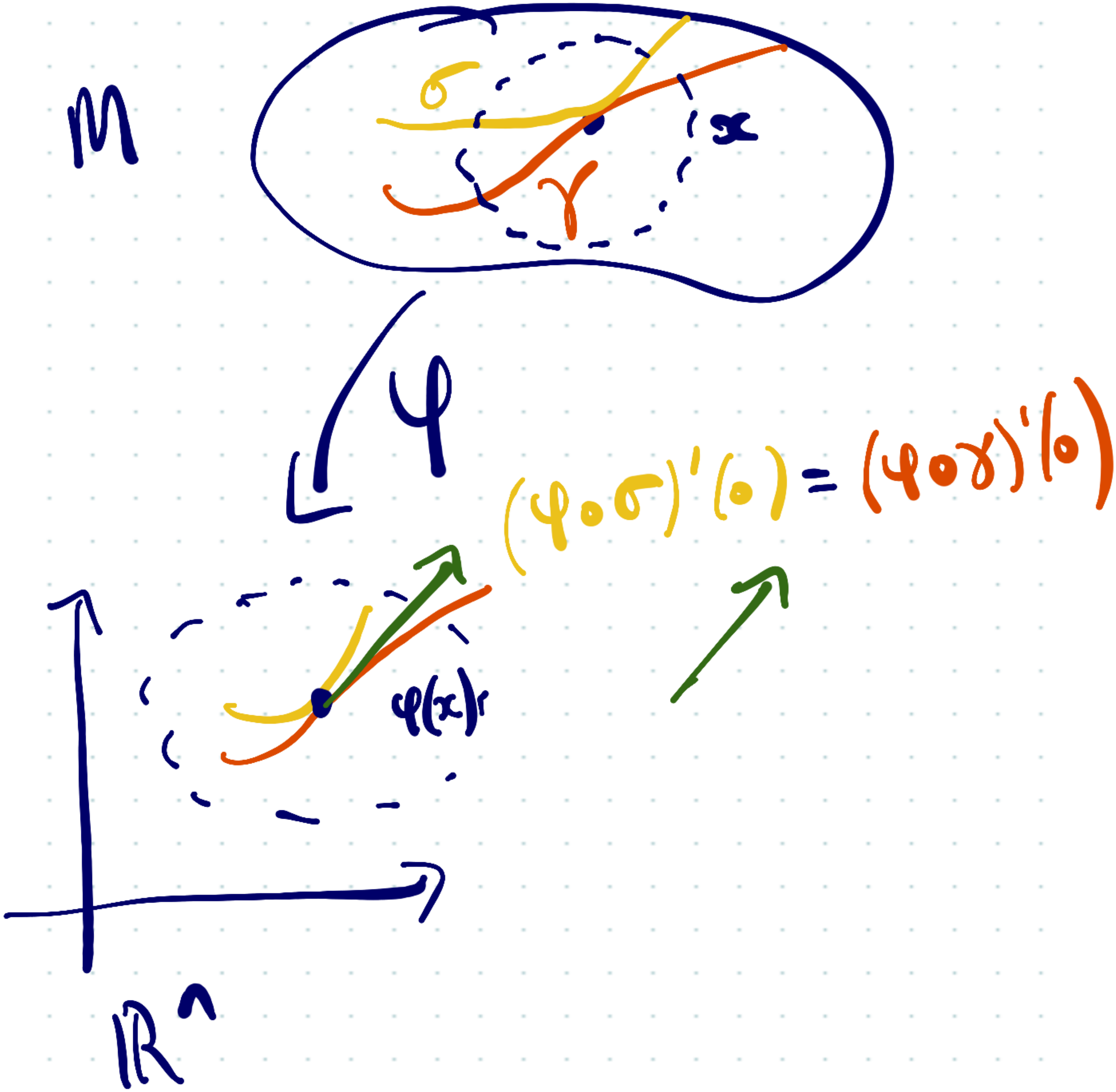
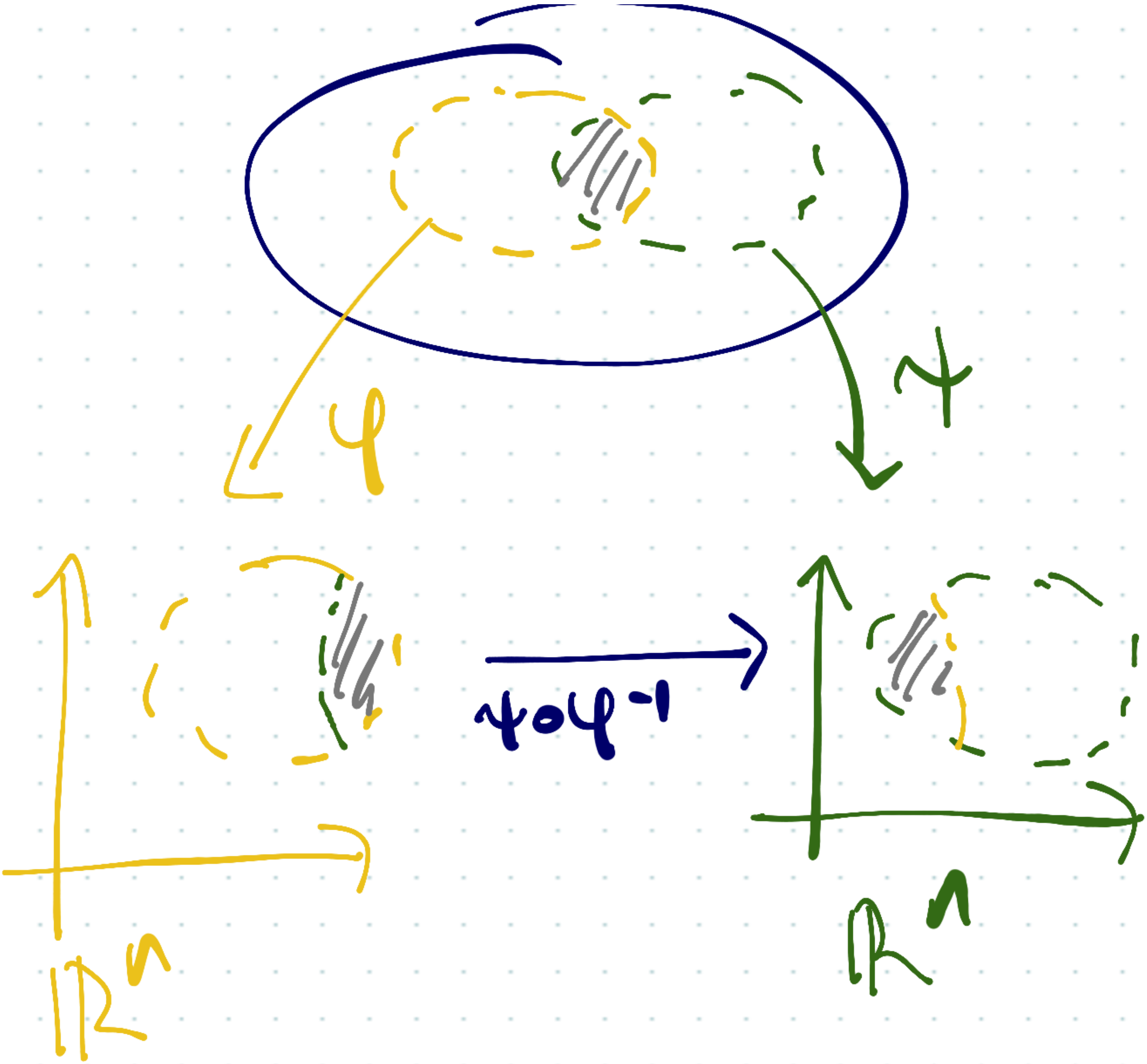
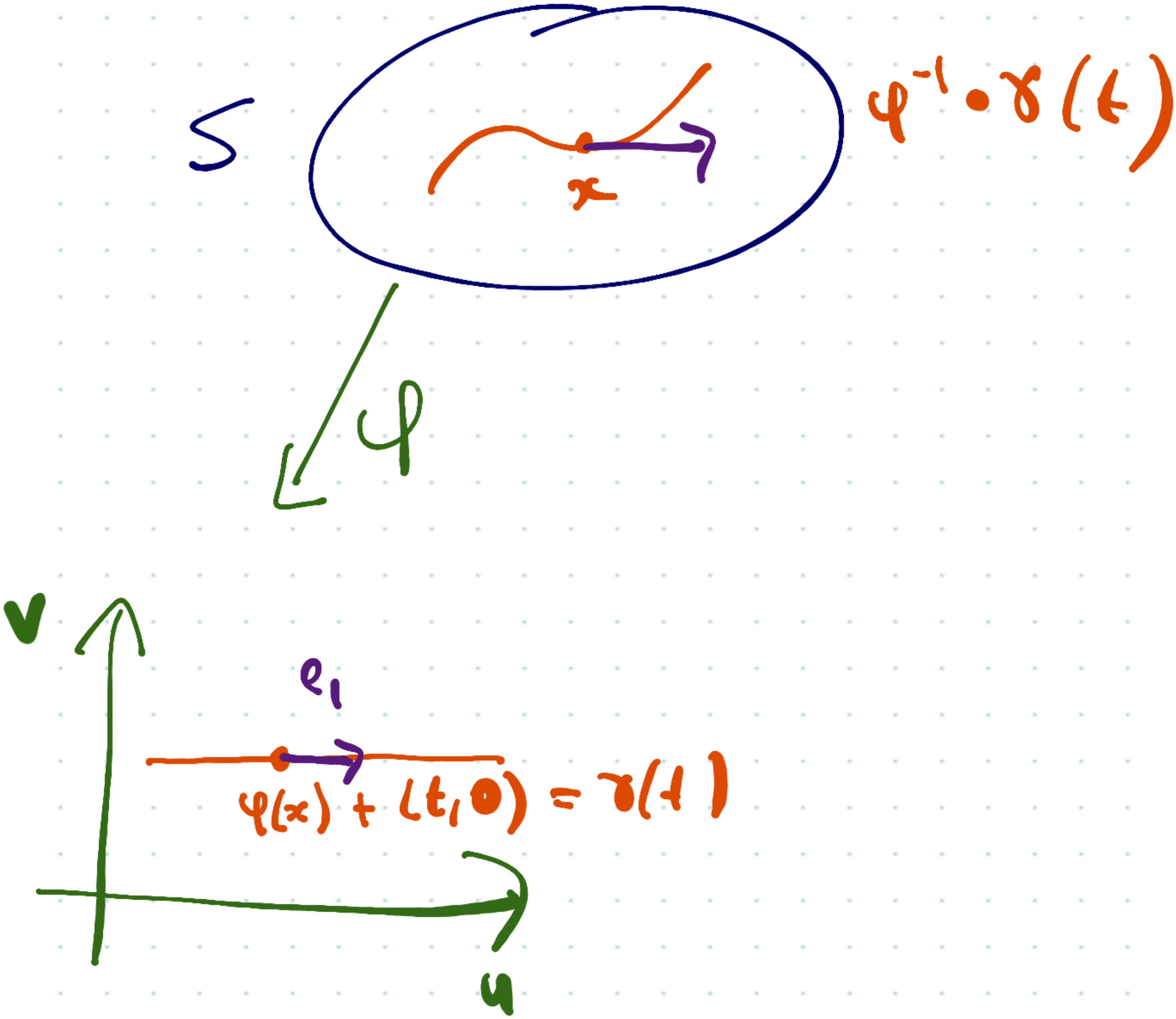
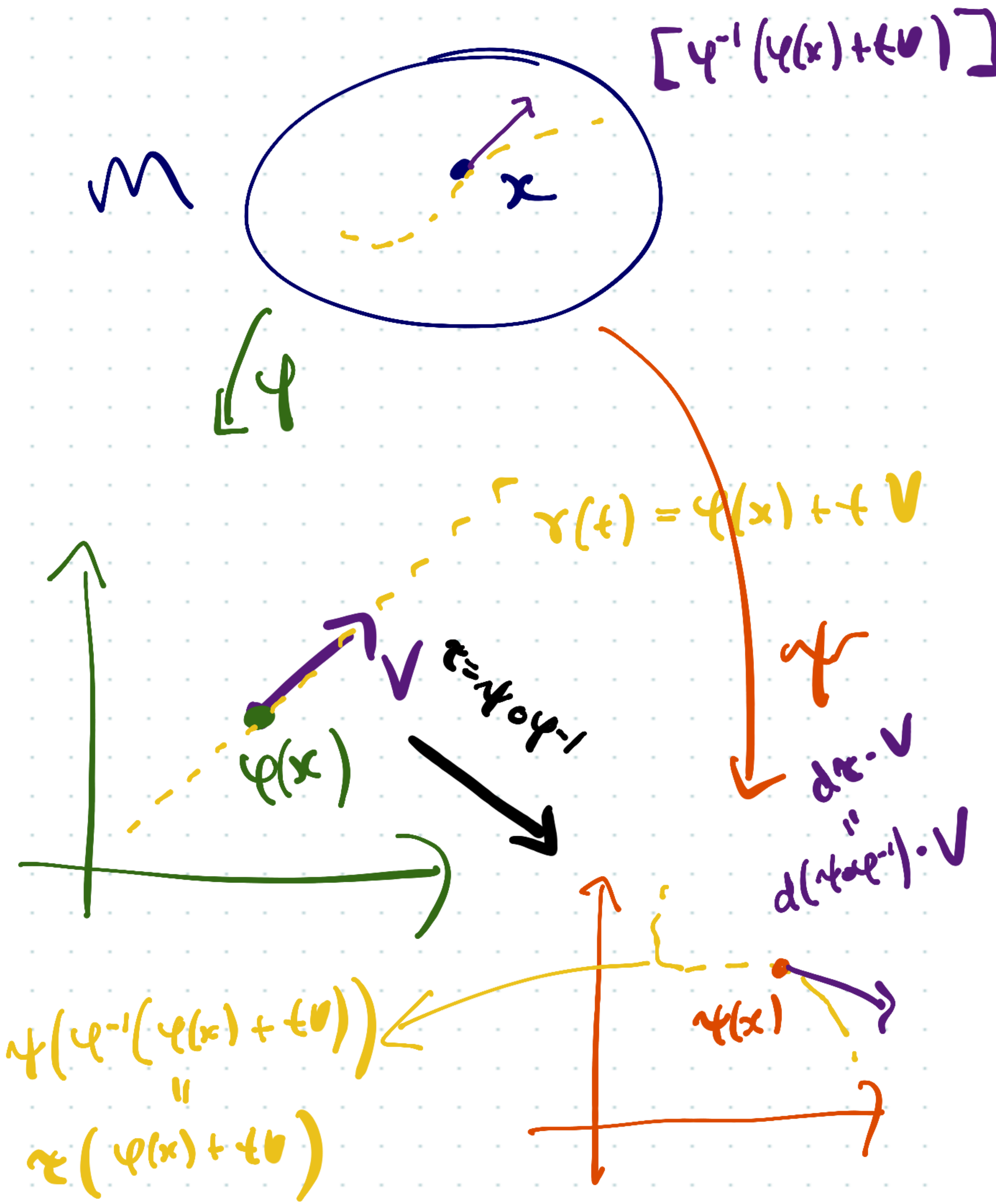

M
σ
x
γ
φ
$(\varphi \circ \sigma)'(0) = (\varphi \circ \gamma)'(0)$
φ(x)
$\mathbb{R}^n$



φ
ψ
$\psi \circ \varphi^{-1}$
$\mathbb{R}^n$
$\mathbb{R}^n$



S
$\varphi^{-1}\circ\gamma(t)$
x
φ
v
e_1
$\varphi(x)+(t,0)=\gamma(t)$
u



$[\varphi^{-1}(\varphi(x)+tv)]$
M
x
φ
$\gamma(t) = \varphi(x) + tv$
v
$\varphi(x)$
$\tau = \psi \circ \varphi^{-1}$
ψ
$d\tau \cdot v$
$=$
$d(\psi \circ \varphi^{-1}) \cdot v$
$\psi(x)$
$\psi(\varphi^{-1}(\varphi(x)+tv))$
$=$
$\tau(\varphi(x)+tv)$

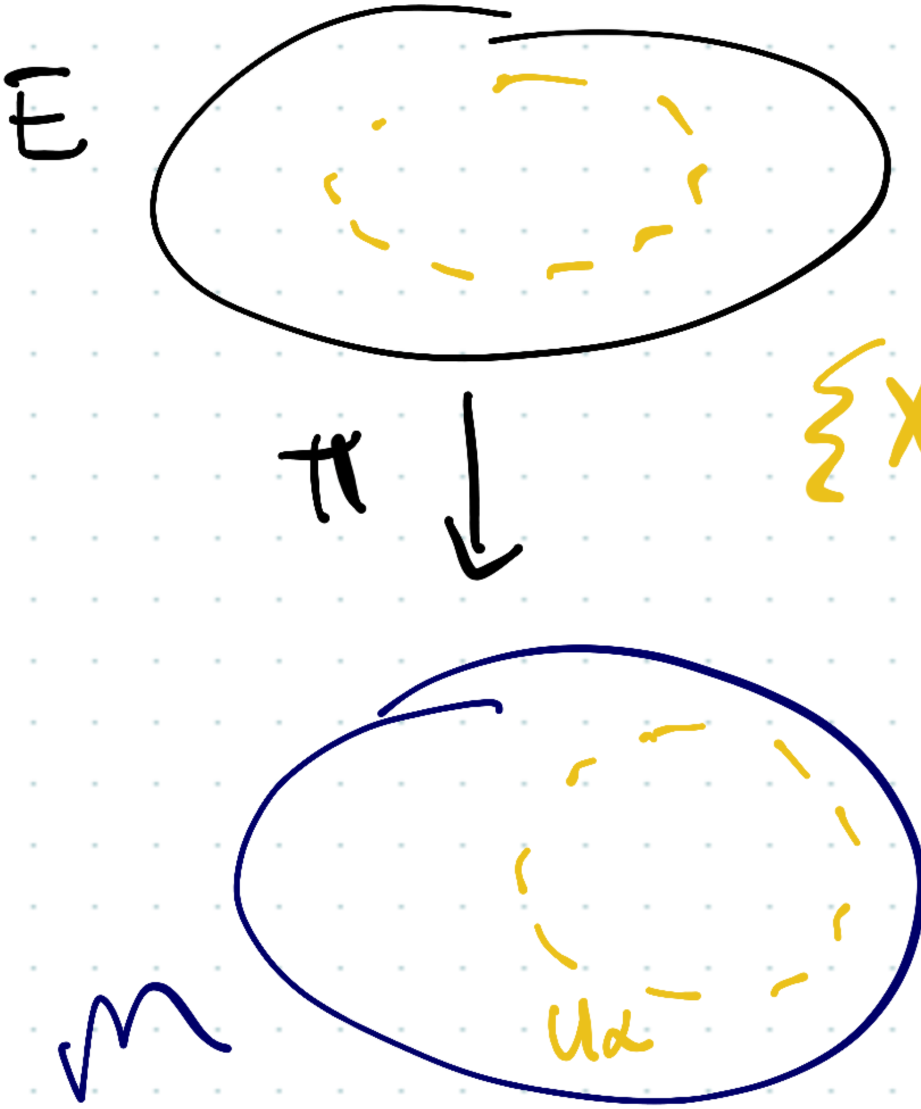
Recall $\tau^{-1} = \varphi \circ \psi^{-1}$

& $\tau \circ \tau^{-1} = Id$

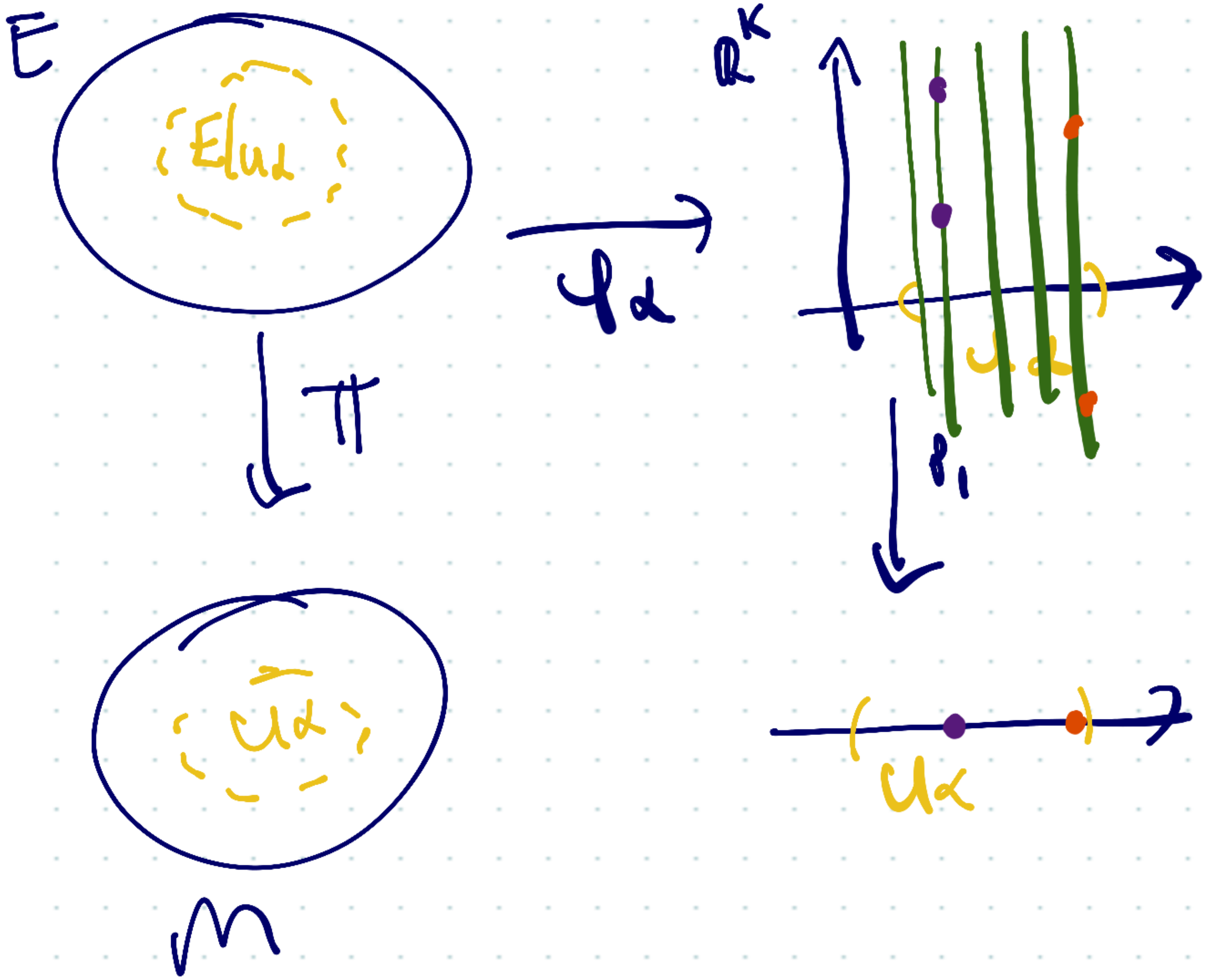
$\therefore \; d\tau \cdot d\tau^{-1} = Id_{n \times n}$

(by chain rule)

i.e. $(d\tau)^{-1} = d\tau^{-1}$



$$E|_{U_\alpha} := \pi^{-1}(U_\alpha) = \{ x \in E : \pi(x) \in U_\alpha \}$$

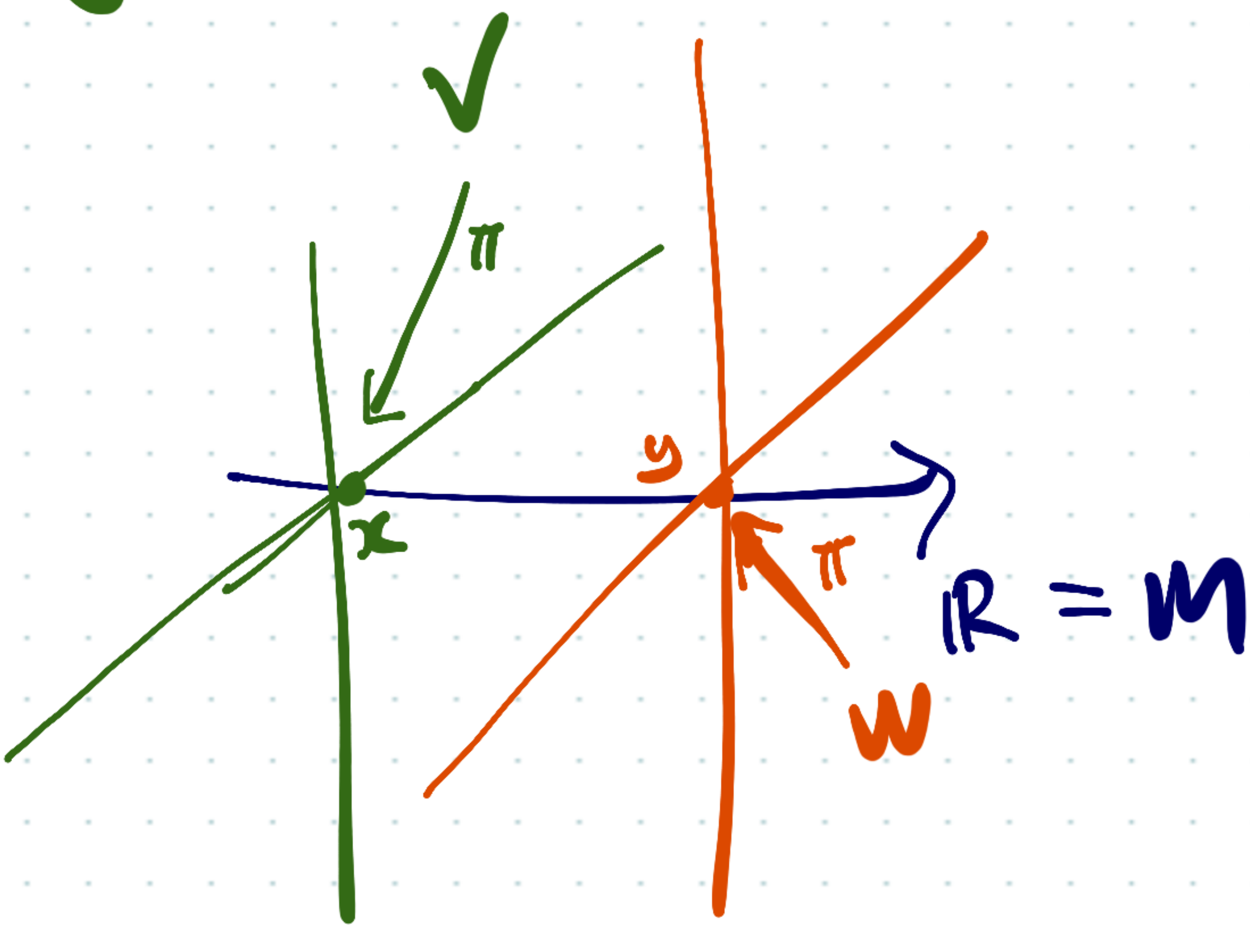

E
E|Uα
φα
ℝ^k
Uα
π
p1
M
Uα
Uα

$x \in \mathbb{R} = M$

$y \in \mathbb{R} = M$

$M = \mathbb{R}$

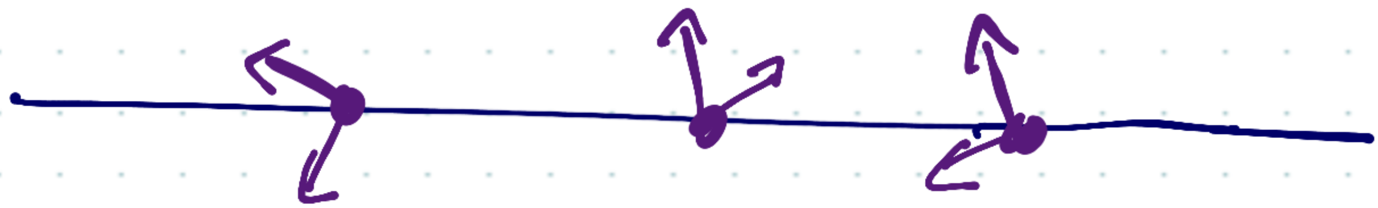
$K = 2$



$$E = \mathbb{R} \times \mathbb{R}^2$$

$(x, V) \in E \qquad \pi(x, V) = x$

$(y, W) \in E \qquad \pi(y, W) = y$



If $X_1, X_2 \in E$

$$(x, V_1^\alpha) = \varphi_\alpha(X_1)$$

$$(x, V_2^\alpha) = \varphi_\alpha(X_2)$$

$$(x, V_1^\beta) = \varphi_\beta(X_1)$$

$$(x, V_2^\beta) = \varphi_\beta(X_2)$$

want

$$\varphi_\alpha^{-1}(x, c^1 V_1^\alpha + c^2 V_2^\alpha)$$

$$= \varphi_\beta^{-1}(x, c^1 V_1^\beta + c^2 V_2^\beta)$$

we showed

$$\tau_{\alpha\beta}\left(x, c^1 v_1^\alpha + c^2 v_2^\alpha\right) \quad (1)$$

$$\tau_{\alpha\beta} = \varphi_\beta \circ \varphi_\alpha^{-1}$$

$$\|$$

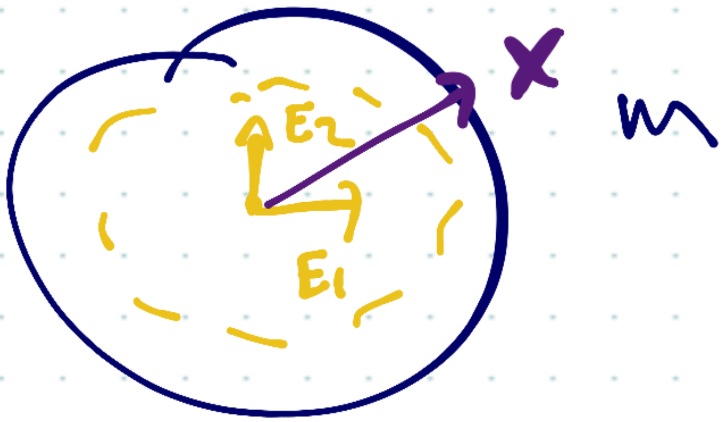
$$\left(x, c^1 v_1^\beta + c^2 v_2^\beta\right) \quad (2)$$

Apply φ_β^{-1} to (1) & (2)

$$\varphi_\alpha^{-1}\left(x, c^1 v_1^\alpha + c^2 v_2^\alpha\right)$$

$$\|$$

$$\varphi_\beta^{-1}\left(x, c^1 v_1^\beta + c^2 v_2^\beta\right)$$



$$X = x^1 e_1 + x^2 e_2$$

$$\Phi_\alpha(X) = (\pi(X), X^1, \dots, X^n)$$

$$p_1 \circ \Phi_\alpha(X) = \pi(X)$$

Define

$$\mathbb{R}^n \times \mathbb{R}^n \longrightarrow T\mathbb{R}^n$$

$$(x, v) \longmapsto [x + tv]$$

note $\forall X \in T\mathbb{R}^n$

$$X = [\gamma] = [\gamma(0) + t\gamma'(0)]$$